Constraints on supersymmetric theories from $\mu \rightarrow e, \gamma$ [*]

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Abstract

In the absence of any additional assumption it is natural to conjecture that sizeable flavour-mixing mass entries, Δm^2, may appear in the mass matrices of the scalars of the MSSM, i.e. $\Delta m^2 \sim O(m^2)$. This flavour violation can still be reconciled with the experiment if the gaugino mass, $M_{1/2}$, is large enough, leading to a *gaugino dominance* framework (i.e. $M_{1/2}^2 \gg m^2$), which permits a remarkably model–independent analysis. We study this possibility focussing our attention on the $\mu \rightarrow e, \gamma$ decay. In this way we obtain very strong and general constraints, in particular $\frac{M_{1/2}^2}{\Delta m} \gtrsim 34$ TeV. On the other hand, we show that our analysis and results remain valid for values of m^2 much larger than Δm^2, namely for $\frac{\Delta m^2}{m^2} \gtrsim \frac{m^2}{10 \text{ TeV}^2}$, thus extending enormously their scope of application. Finally, we discuss the implications for superstring scenarios.

1 Introduction

It is well-known that FCNC processes are very sensitive tests to physics beyond the standard model (SM) and, in particular, to supersymmetric extensions of the SM (SSM). There are two main sources of FCNC signals in the SSM. First, since FCNC processes occur only beyond tree-level, they are sensitive to the existence of new particles circulating in the relevant loops. In this way, the usual SM predictions on FCNC processes (e.g. $K\bar{K}$ mixing or $b \to s, \gamma$) are modified by any SSM [1]. Second, besides the Kobayashi–Maskawa mechanism, supersymmetry provides new direct sources of flavour violation, namely the possible (and even natural as we will see) presence of *off-diagonal* terms (say generically Δm^2) in the squark and slepton mass matrices. In the present paper we will concentrate on this second source of flavour violation, which induces not only modifications in the SM FCNC processes (such as $b \to s, \gamma$), but also the appearance of new FCNC processes, particularly in the leptonic sector, e.g. $\mu \to e, \gamma$ or $\tau \to e, \gamma$.

In this paper we will focus all our attention on the constraints on Δm^2 from the $\mu \to e, \gamma$ process for the following reasons:

i) Among all the FCNC processes (both in the hadronic and in the leptonic sector) $\mu \to e, \gamma$ is by far the one with higher potential to restrict the value of the off-diagonal terms, Δm^2 [2, 3, 4, 5, 6].

ii) Unlike in the hadronic processes, the calculation of $\mu \to e, \gamma$ is very clean and not affected by big uncertainties.

Actually, we will see that the constraints on the SSM coming from $\mu \to e, \gamma$ are not only very strong, but also that its evaluation is remarkably independent of the particular details of the model under consideration.

Let us briefly review the emergence of non-diagonal scalar mass matrices in the minimal supersymmetric standard model (MSSM).

The MSSM is defined by the superpotential, W, and the form of the soft supersymmetry breaking terms. W is given by

$$W = \sum_{i,j} \left\{ h_{ij}^u Q_i H_2 u_{Rj} + h_{ij}^d Q_i H_1 d_{Rj} + h_{ij}^e L_i H_1 e_{Rj} \right\} + \mu H_1 H_2 \ , \tag{1}$$

where i, j are generation indices, Q_i (L_i) are the scalar partners of the quark (lepton) SU(2) doublets, u_{Ri}, d_{Ri} (e_{Ri}) are the quark (lepton) singlets and $H_{1,2}$ are the two SUSY Higgs doublets; the h_{ij}–factors are the (matricial)Yukawa couplings and μ is the usual Higgs mixing parameter. In all terms of eq. (1) the usual SU(2) contraction is assumed, e.g. $H_1 H_2 \equiv \epsilon_{\alpha\beta} H_1^\alpha H_2^\beta$ with $\epsilon_{12} = -\epsilon_{21} = 1$. From W the (global) supersymmetric part of the Lagrangian, $\mathcal{L}_{\text{SUSY}}$, is readily obtained.

$$\begin{aligned} \mathcal{L}_{\text{SUSY}} &= -\sum_k \left| \frac{\partial W}{\partial \phi_k} \right|^2 - \frac{1}{2} \left(\sum_{k,l} \left[\frac{\partial^2 W}{\partial \phi_k \partial \phi_l} \right] \psi_k \psi_l + \text{h.c.} \right) + \text{D} - \text{terms} \\ &+ \text{gauge interactions} \ , \end{aligned} \tag{2}$$

where $\phi_{k,l}$ ($\psi_{k,l}$) run over all the scalar (fermionic) components of the chiral superfields.

In addition to this, the soft breaking terms (gaugino and scalar masses, and tri-linear and bilinear scalar terms) coming from the (unknown) supersymmetry breaking mechanism have the form

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}} \;=\;& \frac{1}{2} M_a \lambda_a \lambda_a + \left(m_L^2\right)_{ij} \bar{L}_i L_j \;+\; \left(m_{e_R}^2\right)_{ij} \bar{e}_{Ri} e_{Rj} \\
+\;& \left(m_Q^2\right)_{ij} \bar{Q}_i Q_j \;+\; \left(m_{u_R}^2\right)_{ij} \bar{u}_{Ri} u_{Rj} \;+\; \left(m_{d_R}^2\right)_{ij} \bar{d}_{Ri} d_{Rj} \\
+\;& \left[A_{ij}^u h_{ij}^u Q_i H_2 u_{Rj} + A_{ij}^d h_{ij}^d Q_i H_1 d_{Rj} + A_{ij}^e h_{ij}^e L_i H_1 e_{Rj} + B\mu H_1 H_2 + \text{h.c.} \right] ,
\end{aligned}
\tag{3}
$$

where a is a gauge group index, λ_a are the gauginos, and the remaining fields in the formula denote just their corresponding scalar components. In the simplest version of the MSSM the soft breaking parameters are taken as *universal* (at the unification scale M_X). Then, the independent parameters of the theory are

$$
\mu, m, M_{1/2}, A, B
\tag{4}
$$

(the rest of the parameters can be worked out demanding a correct unification of the gauge coupling constants and correct masses for all the observed particles). However this simplification is not at all a general principle. In particular there is no theoretical argument against m_{ij}^2, A_{ij} having a non-diagonal structure (as reflected in eq. (3)). The ultimate reason for the specific pattern of these matrices has to be searched in the type of SUGRA theory from which the MSSM derives (particularly in the structure of the Kähler potential) and on the mechanism of supersymmetry breaking. Both ingredients are nowadays unknown, although superstring theories offer plausible ansatzs, which, by the way, do not support in general the universality assumption (we will turn to this point later). On the other hand, it is expected that the Yukawa matrices, h_{ij}^u, h_{ij}^d, h_{ij}^e, may be also non-diagonal (this is actually obliged for h_{ij}^u or h_{ij}^d). After performing the usual rotation in the superfields to diagonalize[1] h_{ij}^u, h_{ij}^d, h_{ij}^e, the fermionic mass matrices and the contribution to the scalar mass matrices coming from[2] (2) become diagonal. However, if the m_{ij}^2 matrices are not universal, they will develop off-diagonal entries, even if they are initially diagonal. A similar thing occurs with the A_{ij} matrices. In the following we will concentrate our attention on the $(m_{ij}^2)_{i\neq j} \equiv \Delta m_{ij}^2$ terms, setting $(A_{ij})_{i\neq j} = 0$. Notice here that A_{ij} always comes accompanied by the corresponding Yukawa coupling (see eq. (3) or eqs. (9,10) below). This makes its effect negligible when studying the impact of Δm_{ij}^2 on $\mu \to e, \gamma$. On the other, hand $(A_{ij})_{i\neq j}$ is also constrained by FCNC processes [2, 4] (and in particular by $\mu \to e, \gamma$ itself), but we will not be concerned by this issue in the present work.

From the previous arguments, it is natural to assume that the off-diagonal entries, Δm^2, can be *sizeable*, or even of the same order as the diagonal terms

$$
\Delta m^2 \sim O(m^2) .
\tag{5}
$$

Certainly, there are proposed mechanisms to avoid this, e.g. the above-mentioned assumption of universality, horizontal symmetries [7, 8], plastic soft terms [9], etc. On the other hand, in some string scenarios [10, 11, 12] Δm^2 could naturally be very small.

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[1]We will work from now on in this new basis for the superfields.
[2]This contribution only takes place once $\langle H_1 \rangle$, $\langle H_2 \rangle \neq 0$ and, except for the stop mass, is much smaller than the mass terms of eq. (3).

However one should wonder whether, in the absence of any additional assumption, the perfectly possible and even natural situation of eq. (5) could still be compatible with the experimental data and, more precisely, with the present experimental bound [13] on $\mu \to e, \gamma$

$$BR(\mu \to e, \gamma) \leq 5 \times 10^{-11} \ , \tag{6}$$

which, as stated above, gives the strongest constraints on Δm^2, more precisely on $\Delta m_{e\mu}^2$

This will be the main goal of this article. The results will turn out to be surprisingly model-independent. In fact they will depend only on two parameters: the value of Δm^2 itself and the gaugino mass, $M_{1/2}$. Also, we will make more precise (and relaxed) the assumption of eq. (5), namely our results will be valid for any Δm^2 satisfying

$$\frac{\Delta m^2}{m^2} \gtrsim \frac{m^2}{10 \text{ TeV}^2} \ . \tag{7}$$

In principle it might seem that $\Delta m_{e\mu}^2 = O(m_{e,\mu}^2)$ is absolutely incompatible with (6) due to the existence of diagrams of the type of those shown in Fig. 1 (to be discussed in detail in sect. 3). However this is not necessarily the case, since $\mu \to e, \gamma$ is a low energy process, while the assumption (5) naturally arises at some unification scale, $\sim M_X$. Hence, an ordinary renormalization group (RG) running is in order. As a matter of fact, this running softens the problem, since the diagonal entries of the mass matrices increase substantially [14, 12] while the off–diagonal ones remain almost unchanged. This possibility has also been considered in ref. [4], though the calculation was affected by very large uncertainties, as we will see afterwards.

In sect. 2 the RG running from high to low energies is carried out, stressing the fact that the $\Delta m^2 \sim O(m^2)$ conjecture, eq. (5), automatically leads to a "gaugino dominance" framework (i.e. $M_{1/2}^2 \gg m^2$), where all the relevant low-energy quantities apart from Δm^2 itself are determined in terms of a unique parameter, $M_{1/2}$. In sect. 3 the complete calculation of $BR(\mu \to e, \gamma)$ is performed, indicating which are the most important diagrams and why. The results are presented in sect. 4. They are very general, depending only on the values of $M_{1/2}$ and Δm, and have strong implications, in particular the need of a large and precise $M_{1/2}^2/\Delta m^2$ hierarchy in order to maintain the consistency with the experiment. Furthermore, it will be shown that the analysis and results are valid for a wide range of $\Delta m^2/m^2$ values, as anticipated in eq. (7). Finally, the summary and conclusions are presented in sect. 5.

2   From high to low energies

The expression of $BR(\mu \to e, \gamma)$ in the MSSM depends on several low-energy parameters, namely $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$, μ, A, and the spectrum of masses of sleptons and gauginos (the details of the precise dependence are left for sect. 3). These are not independent parameters. They come as a low energy result of the actual initial parameters of the theory (see eq. (4)), which are assumed to be given at the unification scale, M_X. Of course, the connection has to be made through the corresponding RGEs. Next we list some of the most relevant RGEs in our calculation [15]:

- Gaugino masses:

$$\frac{dM_a}{dt} = -b_a \tilde{\alpha}_a M_a \ , \quad a = 1, 2, 3 \tag{8}$$

where $t = 2\log(M_X/Q)$, Q being the scale at which we evaluate the masses, b_a are the coefficients of the 1–loop beta functions for the gauge couplings, $\tilde{\alpha}_a = \alpha_a/(4\pi)$, and α_a are the gauge coupling constants.

- Scalar mass matrices for sleptons:

$$
\begin{aligned}
\frac{d(m_L^2)_{ij}}{dt} &= \delta_{ij}\left(3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2\right) - \frac{1}{16\pi^2}\left[\frac{1}{2}(h^e h^{e\dagger} m_L^2)_{ij} + \frac{1}{2}(m_L^2 h^e h^{e\dagger})_{ij}\right.\\
&+ \left.(h^e m_{e_R}^2 h^{e\dagger})_{ij} + (m_{H_1}^2 h^e h^{e\dagger})_{ij} + (\hat{A}^e \hat{A}^{e\dagger})_{ij}\right] ,\\
\frac{d(m_{e_R}^2)_{ij}}{dt} &= \delta_{ij}\left(4\tilde{\alpha}_1 M_1^2\right) - \frac{1}{16\pi^2}\left[(h^{e\dagger} h^e m_{e_R}^2)_{ij} + (m_{e_R}^2 h^{e\dagger} h^e)_{ij}\right.\\
&+ \left.2\left((h^{e\dagger} m_L^2 h^e)_{ij} + (m_{H_1}^2 h^{e\dagger} h^e)_{ij} + (\hat{A}^{e\dagger}\hat{A}^e)_{ij}\right)\right] ,
\end{aligned} \tag{9}
$$

where $(\hat{A}^{u,d,e})_{ij} = (A^{u,d,e})_{ij}(h^{u,d,e})_{ij}$.

- Matricial trilinear terms (also for sleptons):

$$
\begin{aligned}
\frac{d(A^e)_{ij}}{dt} &= -\delta_{ij}\left(3\tilde{\alpha}_2 M_2 + 3\tilde{\alpha}_1 M_1 + \frac{1}{16\pi^2}\left(\mathrm{Tr}(\hat{A}^e h^{e\dagger}) + 3\mathrm{Tr}(\hat{A}^d h^{d\dagger})\right)\right)\\
&- \frac{1}{16\pi^2}\left[2(\hat{A}^e h^{e\dagger})_{ij} + (h^{e\dagger}\hat{A}^e)_{ij}\right] .
\end{aligned} \tag{10}
$$

- μ parameter:

$$
\frac{d\mu^2}{dt} = (3\tilde{\alpha}_2 + \tilde{\alpha}_1)\mu^2 - \frac{1}{16\pi^2}\mathrm{Tr}\left(3h^u h^{u\dagger} + 3h^d h^{d\dagger} + h^e h^{e\dagger}\right)\mu^2 . \tag{11}
$$

- B parameter:

$$
\begin{aligned}
\frac{dB}{dt} &= -(3\tilde{\alpha}_2 M_2 + \tilde{\alpha}_1 M_1) - \frac{1}{16\pi^2}\mathrm{Tr}\left(3(h^u \hat{A}^{u\dagger})\right.\\
&+ \left.3(h^d \hat{A}^{d\dagger}) + (h^e \hat{A}^{e\dagger})\right) .
\end{aligned} \tag{12}
$$

The first thing to notice is that, due to the structure of the equations for scalar masses (9), the ratio $\Delta m^2/m^2$ will in general be small at low energies (even if it is $O(1)$ at M_X), provided that gaugino masses are bigger[3] than scalar masses, $M_{1/2}^2 \gg m^2$, because of the contribution of the former in the RGEs of the diagonal parts of the latter, which is not the case for the off-diagonal entries (note the δ_{ij} factors in eqs. (9)).

This is the reason why the RGEs have the potential to "cure" initial sizeable values of Δm. Therefore, the assumption of naturally large flavour mixing at M_X (see eq. (5)) leads us necessarily to the $M_{1/2}^2 \gg m^2$ scenario. Moreover, this sometimes called "gaugino dominance" scenario [14] presents other very interesting features, such as that *all* the soft breaking parameters (M_a, m_{ij}^2, A_{ij}^u, A_{ij}^d, A_{ij}^e and B) are essentially determined *at low energies* by the value of $M_{1/2}$ at M_X, independently of their initial values, as can be again easily seen in eqs. (8, 9, 10, 12). So it is perfectly sensible

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[3]An evaluation of the hierarchy which is needed between M and m will be given as a result of the whole calculation in the following sections.

to set the ms, As and B equal to zero[4], and this is what we shall do for the rest of the analysis. Note that all this does *not* apply to the μ parameter as it renormalises proportional to itself (see eq. (11)). So, for a given $M_{1/2}^2 \gg m^2$, the only parameter left from the initial set (4) to determine the whole low energy SUSY spectrum is μ.

However there is a constraint which we have not considered up to now, namely the requirement of a correct electroweak breaking. This fixes the value of μ, giving us the *whole* spectrum and other relevant low-energy quantities (such as A and $\tan\beta$) in terms of a *unique* parameter $M_{1/2}$. From the technical point of view, we have performed the calculation along the lines already sketched in a previous work [17], that is, we have computed the 2–loop RGEs for the gauge couplings and imposed their unification at M_X, taking into account all the SUSY thresholds. Also, to obtain a correct electroweak breaking, we have included all the spectrum in the 1-loop effective potential for the Higgs fields[5], choosing the appropriate minimization scale as was already explained in [18, 19]. In this task (and also in the rest of the paper) we work under the assumption that only the Yukawa couplings of the third generation are the relevant ones, and their values are fixed by the experimental masses of the corresponding fermions (for the top we take $m_t = 175$ GeV).

The results are summarized in Figs. 2a, 2b. As a general feature, which is worth mentioning, we have found that the values of $\tan\beta$ obtained in this framework tend to be rather large (ranging from 11 to 26 as $M_{1/2}$ increases from 150 GeV to 10 TeV, as can be seen from Fig. 2b); this fact will be very important in our calculation given the presence of a term proportional to $\mu \tan\beta$ in one of the contributions to $BR(\mu \to e, \gamma)$ (see eqs. (17) in sect. 3). We should comment here that previous analyses have ignored the electroweak breaking constraints, just fixing $\tan\beta$ to a particular (low) value [2, 4] and making a certain "average" in some low-energy parameters. The present results indicate that this is an important source of error and uncertainty, which can be avoided by taking into account the electroweak breaking process.

To summarize, a scenario of gaugino dominance (which arises from the assumption of naturally large flavour mixing at high energy) determines the low–energy spectrum and other relevant low-energy quantities such as A, μ, $\tan\beta$, in terms of a unique parameter, $M_{1/2}$. This is a very interesting fact that makes the subsequent analysis rather accurate and model–independent.

3   Evaluation of $BR(\mu \to e, \gamma)$

We follow here a scheme of calculation along the lines of refs. [2, 3]. The effective Lagrangian that describes the $\mu \to e, \gamma$ decay is usually parameterized as

$$\mathcal{L}_{eff} = \frac{1}{2}\bar{\psi}_{\text{electron}}\ \sigma_{\mu\nu}(B_L P_L + B_R P_R)\ \psi_{\text{muon}}F^{\mu\nu}\ , \tag{13}$$

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[4]Note here that if $m^2 \ll M_{1/2}^2$ at M_X, then also $A^2 \ll M_{1/2}^2$, $B^2 \ll M_{1/2}^2$, since A, B are necessarily $O(m)$ in order to avoid dangerous charge and color breaking minima. Besides, the value of B is related to the value of A or m in many supergravity models. For a review of these issues see [16] and references therein.

[5]This minimization process is calculated considering only the diagonal mass terms, given the fact that at the electroweak scale the non-diagonal entries are necessarily small, precisely due to FCNC constraints like those studied in this paper.

where $P_{R,L} = (1 \pm \gamma_5)/2$. Then, the branching ratio $BR(\mu \to e, \gamma)$ is given by

$$BR(\mu \to e, \gamma) = \frac{12\pi^2}{G_F^2 m_\mu^2} \left[|B_L|^2 + |B_R|^2 \right] \quad , \tag{14}$$

where G_F is the Fermi constant and m_μ is the muon mass. The B_L, B_R amplitudes arise from one–loop diagrams that involve a flip of the leptonic flavour triggered by the slepton mixing, besides the propagation of a neutralino or chargino (see Fig. 1). Note that the structure of (13) implies that the helicity of the muon and the electron must be opposite. Since the electron and muon Yukawa couplings are very suppressed, only the gauge part of the couplings of the charginos and neutralinos will play a role in the diagrams. In fact, one important consequence of the gaugino dominance framework is that for large enough values of $M_{1/2}$ the neutralinos (charginos) are almost pure neutral (charged) gaugino and higgsino. This result follows from the low-energy inequality $M_a^2, \mu^2 \gg M_W^2$, something that occurs whenever $M_{1/2} \gtrsim 300$ GeV. Then, the relevant diagrams, corresponding to bino ($\tilde{B}$) and wino ($\tilde{W}^0$, $\tilde{W}^-$) exchange [6], are shown in Fig. 1. Some comments are in order here. The crosses in the scalar propagators of the diagrams denote mass insertions. These can be either of the $\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2$, $\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2$, $\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2$ type or of the $\Delta m_{\tilde{\mu}_L \tilde{\mu}_R}^2$ type. The former occur in all the diagrams and correspond to the off-diagonal entries in the soft mass matrices of the sleptons. The latter, occurring only in Diags. 4,6, denotes a change of chirality without flavour mixing. This arises from $\mathcal{L}_{\text{SUSY}}$ and $\mathcal{L}_{\text{soft}}$ (see eqs. (2,3)) once H_1 and H_2 acquire non-vanishing VEVs. It can be checked that $\Delta m_{\tilde{\mu}_L \tilde{\mu}_R}^2 = m_\mu (A^\mu + \mu \tan\beta)$ (note that it is proportional to the fermion mass, which makes $\Delta m_{\tilde{e}_L \tilde{e}_R}^2$ negligible), where from now on we will drop the superindex μ from A^μ. It should be noted that the diagrams of Fig. 1 correspond to an expansion in Δm^2 (both of the $\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2$, $\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2$, $\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2$ type and of the $\Delta m_{\tilde{\mu}_L \tilde{\mu}_R}^2$ type). This is justified due to the smallness of Δm^2 (of either type) compared to the slepton and gaugino masses. Recall here that even though Δm^2 may be $O(m^2)$ at M_X, it must necessarily be $\ll m^2$ at low energies in order to be consistent with the experiment (this will become apparent in the next section). On the other hand the crosses in the fermionic propagators denote a change in helicity, which gives a factor proportional to the fermion mass. Let us remark here that this emerges from the pure calculation and does not amount to any mass expansion.

Splitting the induced $B_{L,R}$ values as

$$\begin{aligned} B_R &= B_R^{\tilde{W}^-} + B_R^{\tilde{W}^0} + B_R^{\tilde{B}} \,, \\ B_L &= B_L^{\tilde{B}} \,, \end{aligned} \tag{15}$$

the different diagrams contribute in the following way:

$$\begin{aligned} B_R^{\tilde{W}^-} &= \mathcal{D}_1 \,, & B_R^{\tilde{W}^0} &= \mathcal{D}_2 \,, \\ B_R^{\tilde{B}} &= \mathcal{D}_3 + \mathcal{D}_4 \,, & B_L^{\tilde{B}} &= \mathcal{D}_5 + \mathcal{D}_6 \,, \end{aligned} \tag{16}$$

where $\mathcal{D}_1 - \mathcal{D}_6$ correspond to diagrams 1–6 of Fig. 1. We have evaluated *all* of them. The expressions given in the previous literature are either incomplete or not directly

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[6]Due to the above-mentioned smallness of the electron and muon Yukawa couplings, we have neglected the diagrams involving higgsinos. Notice also that we have not included contributions where the helicity flip takes place on the outgoing electron, because they are suppressed by a m_e/m_μ factor.

applicable to our case. Some of them correspond to the limit in which the photino exchange gives the main contribution (which is not valid in this case). Other estimates do not take into account the left-right mixing between charged sleptons, that will be essential, or ignore some of the diagrams. Also, it is normally assumed that left and right sleptons are degenerated in mass[7]. However, it is clear that a mass splitting will usually arise due to the different dependence of the RGEs, eq. (9), on the gaugino masses. In the gaugino dominance scenario this effect tends to be very large, see Fig. 2a.

The complete expressions are:

$$\mathcal{D}_1 = \frac{e^3}{16\pi^2} \frac{m_\mu}{m_{\tilde{\nu}}^2} \frac{\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2}{m_{\tilde{\nu}}^2} \frac{G(\frac{M_{\tilde{W}^-}^2}{m_{\tilde{\nu}}^2})}{\sin^2 \theta_W}$$

$$\mathcal{D}_2 = -\frac{e^3}{16\pi^2} \frac{m_\mu}{m_{\tilde{l}_L}^2} \frac{\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2}{m_{\tilde{l}_L}^2} \frac{F(\frac{M_{\tilde{W}^0}^2}{m_{\tilde{l}_L}^2})}{2\sin^2 \theta_W}$$

$$\mathcal{D}_3 = -\frac{e^3}{16\pi^2} \frac{m_\mu}{m_{\tilde{l}_L}^2} \frac{\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2}{m_{\tilde{l}_L}^2} \frac{F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_L}^2})}{2\cos^2 \theta_W}$$

$$\mathcal{D}_4 = \frac{e^3}{16\pi^2} \frac{M_{\tilde{B}} m_\mu (A + \mu \tan\beta)}{(m_{\tilde{l}_L}^2 - m_{\tilde{l}_R}^2)} \left[\frac{\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2}{(m_{\tilde{l}_L}^2 - m_{\tilde{l}_R}^2)} \left(\frac{F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_L}^2})}{m_{\tilde{l}_L}^2} - \frac{F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_R}^2})}{m_{\tilde{l}_R}^2} \right) \right.$$
$$\left. + \frac{\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2}{m_{\tilde{l}_L}^2} \frac{L(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_L}^2})}{m_{\tilde{l}_L}^2} \right] \frac{1}{\cos^2 \theta_W}$$

$$\mathcal{D}_5 = -\frac{e^3}{16\pi^2} \frac{m_\mu}{m_{\tilde{l}_R}^2} \frac{\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2}{m_{\tilde{l}_R}^2} \frac{2F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_R}^2})}{\cos^2 \theta_W}$$

$$\mathcal{D}_6 = \frac{e^3}{16\pi^2} \frac{M_{\tilde{B}} m_\mu (A + \mu \tan\beta)}{(m_{\tilde{l}_R}^2 - m_{\tilde{l}_L}^2)} \left[\frac{\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2}{(m_{\tilde{l}_R}^2 - m_{\tilde{l}_L}^2)} \left(\frac{F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_R}^2})}{m_{\tilde{l}_R}^2} - \frac{F(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_L}^2})}{m_{\tilde{l}_L}^2} \right) \right.$$
$$\left. + \frac{\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2}{m_{\tilde{l}_R}^2} \frac{L(\frac{M_{\tilde{B}}^2}{m_{\tilde{l}_R}^2})}{m_{\tilde{l}_R}^2} \right] \frac{1}{\cos^2 \theta_W} \tag{17}$$

where

$$m_{\tilde{\nu}_e}^2 = m_{\tilde{\nu}_\mu}^2 \equiv m_{\tilde{\nu}}^2 , \quad m_{\tilde{e}_L}^2 = m_{\tilde{\mu}_L}^2 \equiv m_{\tilde{l}_L}^2 , \quad m_{\tilde{e}_R}^2 = m_{\tilde{\mu}_R}^2 \equiv m_{\tilde{l}_R}^2 \tag{18}$$

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[7]These incomplete expressions in the literature should however coincide with a piece of our calculation in the appropriate limits. In this sense, we obtain a complete agreement with ref. [2] while, regarding ref. [3], we find an additional factor 2 in $\mathcal{D}_4$ and a different relative sign between $\mathcal{D}_1$ and $\mathcal{D}_2$ (see eqs. (17) below).

and

$$F(x) = \frac{1}{12}(1-x)^{-5}\left[17x^3 - 9x^2 - 9x + 1 - 6x^2(x+3)\ln x\right]$$

$$G(x) = \frac{1}{6}(1-x)^{-5}\left[-x^3 - 9x^2 + 9x + 6x(x+1)\ln x\right]$$

$$L(x) = \frac{1}{2}(1-x)^{-4}\left[-5x^2 + 4x + 1 + 2x(x+2)\ln x\right] \tag{19}$$

Note that the equalities of eq. (18) do not come from any universality assumption, but from the mere gaugino dominance in the corresponding RGEs.

Although in principle all the diagrams can have a similar magnitude (e.g. if we assume $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} \sim \Delta m^2_{\tilde{e}_L\tilde{\mu}_L} \sim \Delta m^2_{\tilde{e}_R\tilde{\mu}_R}$), in practice diagrams 4 and 6 are the dominant ones. This comes from the coefficient of proportionality $(A + \mu\tan\beta)$, which turns out to be very important in the gaugino dominance framework due to the large $\tan\beta$ value (see sect. 2).

Finally, let us remark that the previous results on $BR(\mu \to e, \gamma)$ are in agreement with the recent paper of ref. [20], where charginos and neutralinos are allowed to be non-pure gaugino or higgsino states.

4   Results

The expressions obtained in the previous section for $BR(\mu \to e, \gamma)$ depend on two different sets of parameters. First, the different masses involved in the game ($m^2_{\tilde{\nu}}$, $m^2_{\tilde{l}_L}$, $m^2_{\tilde{l}_R}$, $M_{\tilde{B}}$, $M_{\tilde{W}}$) and certain relevant low-energy quantities ($A, \mu, \tan\beta$). Second, the three independent flavour-mixing mass entries: $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}$, $\Delta m^2_{\tilde{e}_L\tilde{\mu}_L}$, $\Delta m^2_{\tilde{e}_R\tilde{\mu}_R}$. As explained in sect. 2, once we are working in the framework of gaugino dominance, $M^2_{1/2} \gg m^2$, the first set is completely determined in terms of the initial gaugino mass, $M_{1/2}$. Recall that we were led to this framework by the mere assumption of naturally large flavour mixing at M_X (see eq. (5)). The three flavour-mixing mass parameters, however, remain independent (and we will consider them in that way in the following), although it is logical to suppose that they are of the same order.

The constraints on the MSSM from $BR(\mu \to e, \gamma)$ arise by comparing the above calculations (eqs. (14–19)) with the present experimental bound, eq. (6). We have illustrated this in Fig. 3, where an overall mass-mixing parameter $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} = \Delta m^2_{\tilde{e}_L\tilde{\mu}_L} = \Delta m^2_{\tilde{e}_R\tilde{\mu}_R} \equiv \Delta m^2$ has been taken for simplicity. Then we have plotted $BR(\mu \to e, \gamma)$ vs. $M_{1/2}$ for different values of Δm. From this figure we can derive the maximum allowed value of Δm (or, equivalently, the minimum allowed value of $M_{1/2}/\Delta m$) for each value of $M_{1/2}$. This is represented in Fig. 4 for four different cases: a) $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} = \Delta m^2_{\tilde{e}_L\tilde{\mu}_L} = \Delta m^2_{\tilde{e}_R\tilde{\mu}_R} \equiv \Delta m^2$; b) only $\Delta m^2_{\tilde{e}_R\tilde{\mu}_R} \neq 0$; c) only $\Delta m^2_{\tilde{e}_L\tilde{\mu}_L} \neq 0$ and d) only $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} \neq 0$, which gives a complete picture of the results. Notice that the (d) case is the less restrictive one. This is because, as explained in sect. 3, given the large value of $\tan\beta$ the dominant diagrams are nos. 4 and 6 of Fig. 1, which do not involve $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}$, see eq. (17). On the other hand, the strongest constraint comes from $\Delta m^2_{\tilde{e}_R\tilde{\mu}_R}$. This is because the mass of the right sleptons, $m^2_{\tilde{l}_R}$, is smaller than that of the left ones, $m^2_{\tilde{l}_L}$, as a consequence of their different RG running (note from eq. (9) the different dependence of $m^2_{\tilde{l}_L}$, $m^2_{\tilde{l}_R}$ on the gaugino masses).

The constraints are in general extremely strong. For case *(a)*, which is the most representative one, the corresponding curve can be approximately fitted by the simple constraint[8]

$$\frac{M_{1/2}^2}{\Delta m} \gtrsim 34 \text{ TeV} \tag{20}$$

(similar equations can be written for the curves associated to the other *(b)*, *(c)*, *(d)* cases). Under the assumption of eq. (5), i.e. $\Delta m = O(m)$, the results of Fig. 4 or eq. (20) imply that, indeed, a very large hierarchy between the scalar and gaugino masses is needed in order to reconcile the theoretical and experimental results. This gives full justification to our assumption of a gaugino dominance framework once eq. (5) has been conjectured. For example, for $M_{1/2} \sim 500$ GeV the assumption $\Delta m \sim m$ demands $M_{1/2}/\Delta m > 65$. For smaller values of $M_{1/2}$ the required hierarchy is in fact more severe, whereas too large values of $M_{1/2}$ start to conflict an electroweak breaking process with no fine–tuning [19, 21]. Actually, it is hard to think of a scenario where such a dramatical hierarchy can naturally arise. Consequently, we can conclude at this point that a naturally large flavour mixing, as that conjectured in eq. (5), can hardly be reconciled with the experiment in a natural way.

On the other hand, the results from Fig. 4 and eq. (20) are so extreme that we can relax the assumption $\Delta m \sim m$, allowing for larger values of m relative to Δm without losing the validity of the calculation. The latter is based on the use of the gaugino dominance framework, which is reasonably accurate for $M_{1/2}^2 > O(10)\, m^2$. Then, from (20), it follows that all our results, and eq. (20) itself, are valid whenever we start with a Δm at M_X such that

$$\frac{\Delta m^2}{m^2} \gtrsim \frac{m^2}{10 \text{ TeV}^2} , \tag{21}$$

as was foretold in eq. (7). This result extends enormously the scope of application of our calculation and hence its interest. In this way we see that rather small values of $\Delta m^2/m^2$ at M_X require a gaugino dominance scenario to be cured, thus leading to a complete determination of all the relevant low-energy parameters in terms of $M_{1/2}$ (see sect. 2 and Fig. 2).

It is interesting to compare these results with those obtained in the initial calculation of ref. [4]. There, besides assuming $\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2 \neq 0$, $\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2 = \Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2 = 0$ (i.e. our case *(c)*), several additional assumptions and averages were adopted (some of them already commented in sect. 2). The confrontation of the two calculations is made in Fig. 5, where we have represented $\left(M_{1/2}/\Delta m\right)_{\min}$ vs. $m_{\text{av}} \equiv \sqrt{\frac{1}{2}(m_{\tilde{l}_L}^2 + m_{\tilde{l}_R}^2)}$ to facilitate the comparison with the plots of ref. [4]. Obviously, there is a big difference between their results (that we have been able to reproduce) and ours. This reflects the very large uncertainties in the calculation of ref. [4] (acknowledged by its authors), which are dramatically suppressed once a correct electroweak breaking process is imposed.

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[8]A more accurate form for the constraint is $\frac{M_{1/2}^2}{\Delta m} \geq (9 \text{ TeV})\sqrt{\tan \beta}$, where $\tan \beta$ can be well fitted by $\tan \beta = 16 \left[M_{1/2}(\text{TeV})\right]^{1/5}$, see Fig. 2b. The dependence of $\frac{M_{1/2}^2}{\Delta m}$ on $\sqrt{\tan \beta}$ can be understood from the dependence of the dominant diagrams (nos. 4 and 6 of Fig. 1) on $\tan \beta$, see eqs. (17).

5   Summary and concluding remarks

In the absence of any additional assumption it is natural to conjecture that sizeable flavour-mixing mass entries, Δm^2, may appear in the mass matrices of the scalars of the MSSM, i.e. $\Delta m^2 \sim O(m^2)$. This flavour violation can still be reconciled with the experiment if the gaugino mass, $M_{1/2}$, is large enough to yield (through the renormalization group running) a sufficiently small $\Delta m^2/m^2$ at low energy. We have analyzed in detail this possibility, focussing our attention on the leptonic sector, particularly on the $\mu \to e, \gamma$ decay, which is by far the FCNC process with higher potential to restrict the value of the off-diagonal terms, Δm^2. The results are the following:

1. The $\Delta m^2 \sim O(m^2)$ conjecture automatically leads to a *gaugino dominance* framework (i.e. $M_{1/2}^2 \gg m^2$), where, apart from Δm^2 itself, *all* the relevant low-energy quantities (mass spectrum, A, μ, $\tan\beta$) are determined in terms of a unique parameter, $M_{1/2}$ (see Fig. 2). This makes the subsequent analysis and results remarkably model–independent.

2. The resulting constraints in the MSSM, obtained by comparing the calculated $BR(\mu \to e, \gamma)$ with the experimental bound, are very strong (see Figs. 3, 4). More precisely, assuming $\Delta m_{\tilde\nu_e \tilde\nu_\mu}^2 = \Delta m_{\tilde e_L \tilde\mu_L}^2 = \Delta m_{\tilde e_R \tilde\mu_R}^2 \equiv \Delta m^2$, we arrive at the approximate constraint

$$\frac{M_{1/2}^2}{\Delta m} \gtrsim 34 \text{ TeV} \tag{22}$$

(and similar equations for other cases). This makes, in our opinion, the natural flavour mixing conjecture $\Delta m^2 \sim O(m^2)$ extremely hard to be reconciled with the experiment in a natural way. Hence, $\Delta m/m$ should be small already at the unification scale.

3. The required values of $M_{1/2}/\Delta m$ to be within the experimental limits are so large that the gaugino dominance assumption, which is an essential ingredient of our analysis, remains valid for values of m^2 much larger than Δm^2, namely for $\frac{\Delta m^2}{m^2} \gtrsim \frac{m^2}{10 \text{ TeV}^2}$, thus extending enormously the scope of interest and application of our results.

To perform the previous calculation we have completed the earlier evaluations [2, 3] of $BR(\mu \to e, \gamma)$ (see diagrams of Fig. 1). The results, summarized in eqs. (14–19), are in concordance with the recent results of ref. [20].

We would like to stress the high degree of model–independence of our analysis and results. In fact, in the entire calculation we have only made, for the sake of simplicity, the assumption of universality of gaugino masses at the unification scale. The relaxation of this assumption does not imply any essential conceptual change in the analysis, leading to straightforward modifications in the results of the paper, without affecting the main conclusions above.

Finally, let us comment that the need of starting with small $\Delta m/m$ can be satisfied in some theoretically well-founded scenarios, which become favoured from this point of view. In particular, besides the proposed mechanisms of refs. [7, 8, 9], we would like to stress that many string constructions can be consistent with that requirement. More precisely, in orbifold compactifications schemes [22], which are known to give an

interesting phenomenology [23], the soft SUSY breaking scalar masses always consist of a universal piece plus a contribution proportional to the so-called modular weight (n) associated with the field under consideration [24, 10]. Thus, scalar fields belonging to the same twisted sector of the theory (or to the untwisted one) acquire degenerate soft masses, something that can perfectly occur in many realistic scenarios. A similar thing happens in the so-called dilaton-dominated limit [11, 12]. Other scenarios, however, can produce a larger non-universality of the scalar masses, with potentially dangerous contributions to FCNC processes [5]. In any case, these non-universality effects are to produce non-vanishing off-diagonal terms in the scalar mass matrices once the usual rotation of fields to get diagonal fermionic mass matrices is carried out. The phenomenological viability of these physically relevant scenarios undoubtedly deserves further investigation. Work along these lines is currently in progress [25].

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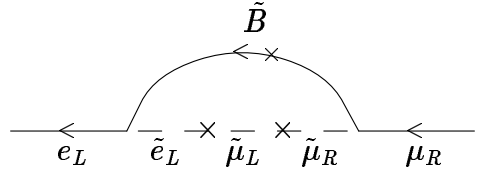[25] B. de Carlos, J.A. Casas and J.M. Moreno, in preparation.
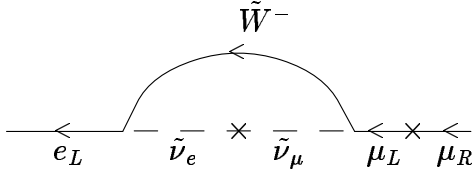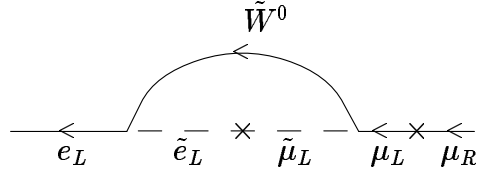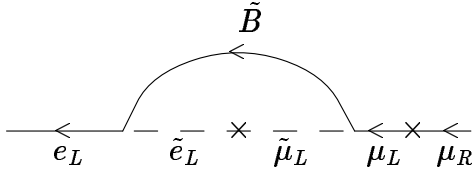
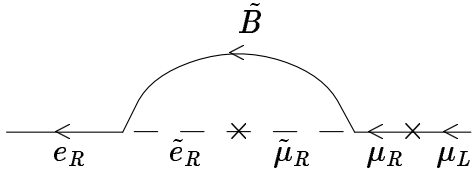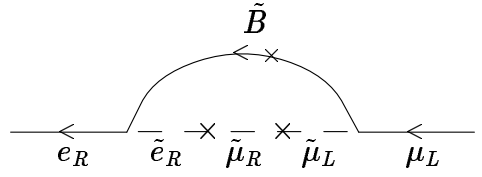Figure 1: Supersymmetric contributions to $\mu \rightarrow e, \gamma$. The crosses indicate mass insertions (for scalars) and helicity flips (for fermions). The outgoing photon (not represented in the graphs) is assumed to be attached to the different diagrams in all possible ways.
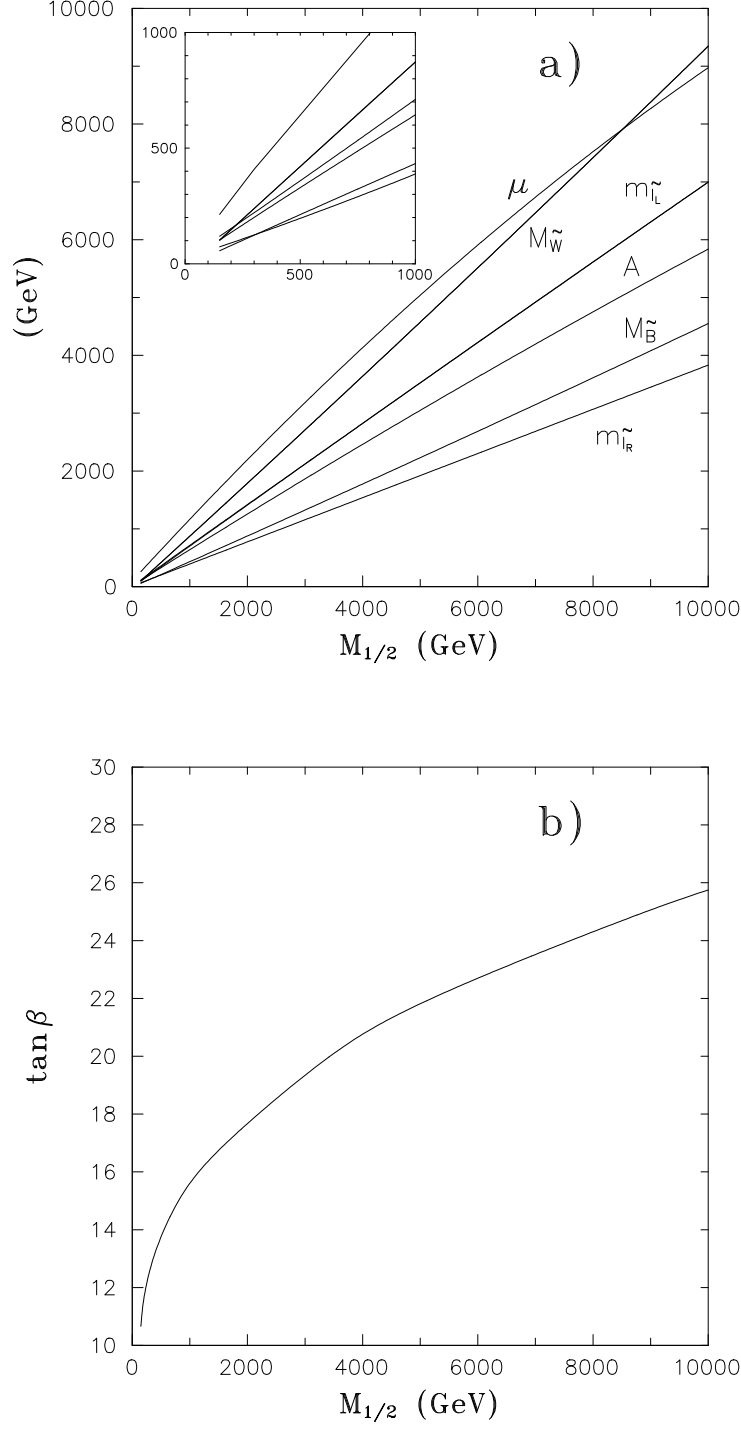
Figure 2: a) Relevant mass spectrum ($M_{\tilde{W}}$, $M_{\tilde{B}}$, $m_{\tilde{l}_L}$, $m_{\tilde{l}_R}$, $m_{\tilde{\nu}}$) and significant low-energy quantities (A, μ) in the gaugino dominance framework ($M_{1/2}^2 \gg m^2$) as functions of $M_{1/2}$ (the only free parameter). The slight non-degeneracies between ($M_{\tilde{W}^0}$, $M_{\tilde{W}^\pm}$) and between ($m_{\tilde{e}_L}$, $m_{\tilde{\nu}_L}$) have not been represented. The curves are cut at $M_{1/2} =$150 GeV, where the spectrum starts to conflict with the experimental bounds. b) Plot of $\tan\beta$ vs. $M_{1/2}$ in the gaugino dominance framework.
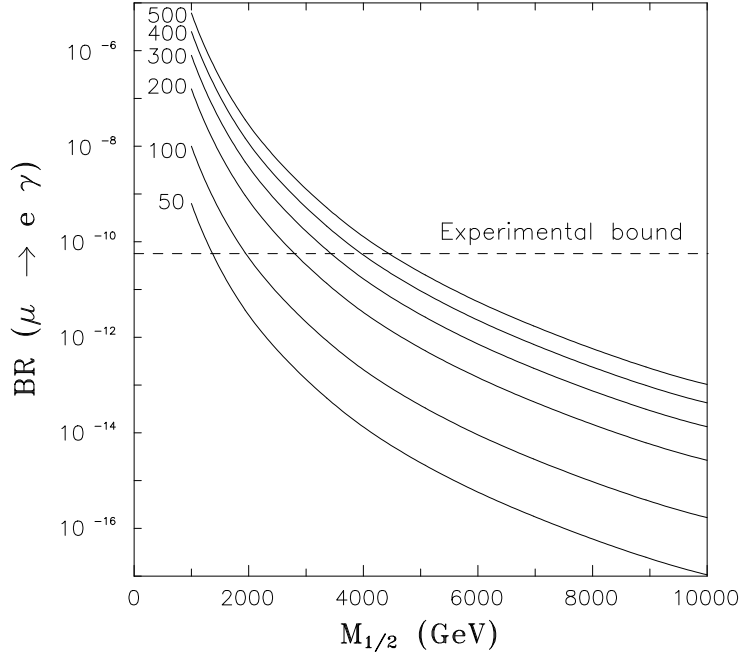
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Figure 3: Plot of $BR(\mu \rightarrow e, \gamma)$ vs. $M_{1/2}$, taking for simplicity $\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2 = \Delta m_{\tilde{e}_L \tilde{\mu}_L}^2 = \Delta m_{\tilde{e}_R \tilde{\mu}_R}^2 \equiv \Delta m^2$. The different curves correspond to $\Delta m = 50$, 100, 200, 300, 400, 500 GeV respectively.
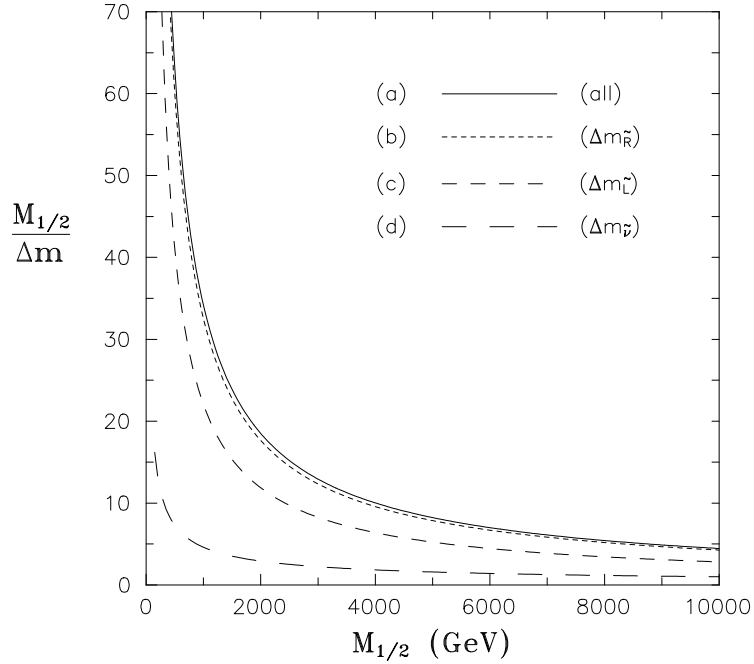

Figure 4: Plot of the *minimum* allowed value of $M_{1/2}/\Delta m$ vs. $M_{1/2}$ in four different cases: *a)* $\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2 = \Delta m_{\tilde{e}_L \tilde{\mu}_L}^2 = \Delta m_{\tilde{e}_R \tilde{\mu}_R}^2 \equiv \Delta m^2$; *b)* only $\Delta m_{\tilde{e}_R \tilde{\mu}_R}^2 \neq 0$; *c)* only $\Delta m_{\tilde{e}_L \tilde{\mu}_L}^2 \neq 0$ and *d)* only $\Delta m_{\tilde{\nu}_e \tilde{\nu}_\mu}^2 \neq 0$.
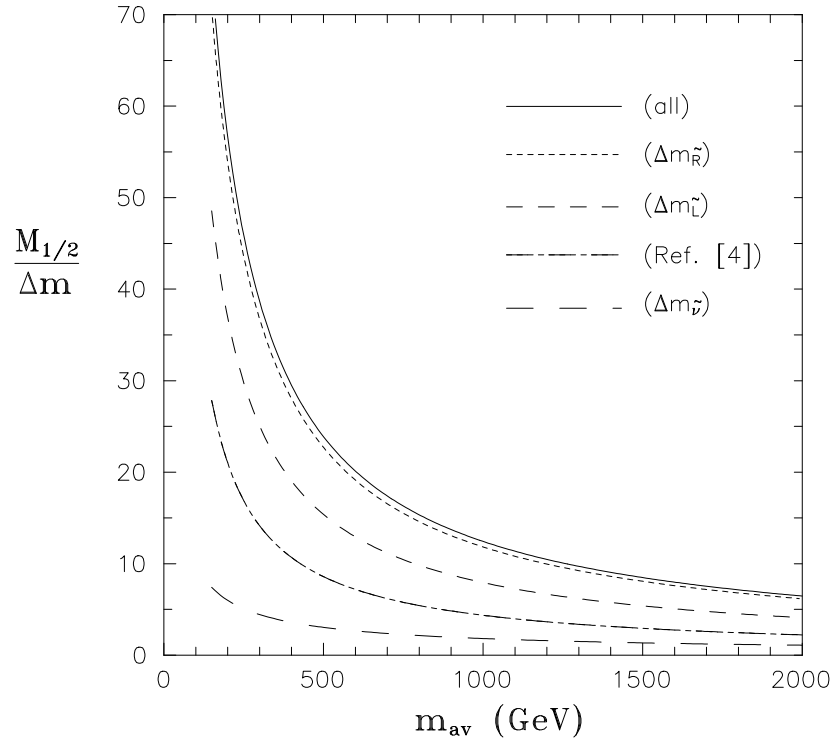
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Figure 5: The same as in Fig. 4, but representing $m_{\rm av} \equiv \left(\frac{1}{2}(m_{\tilde{l}_L}^2 + m_{\tilde{l}_R}^2)\right)^{1/2}$ in the horizontal axis. The dashed–dotted line corresponds to the calculation of ref. [4].